

# Interference Visibility as a Witness of Entanglement and Quantum Correlation

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(Dated: October 31, 2014)

In quantum information and communication one looks for the non-classical features like interference and quantum correlations to harness the true power of composite systems. We show how the concept akin to interference is, in fact, intertwined in a quantitative manner to entanglement and quantum correlation. In particular, we prove that the difference in the squared visibility for a density operator before and after a complete measurement, averaged over all unitary evolutions, is directly related to the quantum correlation measure based on the measurement disturbance. For pure and mixed bipartite states the unitary average of the squared visibility is related to entanglement measure. This may constitute direct detection of entanglement and quantum correlations with quantum interference setups. Furthermore, we prove that for a fixed purity of the subsystem state, there is a complementarity relation between the linear entanglement of formation and the measurement disturbance. This brings out a quantitative difference between two kinds of quantum correlations.

## I. INTRODUCTION

Quantification and detection of quantum entanglement and general quantum correlations are of paramount importance in the field of quantum information. It was already realized in the early days of quantum theory that linear superposition and entanglement are two essential features that distinguishes the quantum world from the classical world [1, 2]. The principle of linear superposition is at the heart of most of the counter intuitive phenomena that we see in quantum mechanics and this also gives rise to quantum entanglement for multipartite pure states. However, for mixed states in addition to entanglement there can be other type of non-classical correlations beyond entanglement. Among mixed state entanglement measures there are many, for e.g. the concurrence [4], entanglement of formation [5, 6], relative entropy of entanglement [7], logarithmic negativity [8] and many more [3, 9]. The non-classical correlations that are not based on the separability paradigm can be quantum discord [10, 11], work deficit [12, 13], quantumness of correlation [14], measurement induced disturbance [15], geometric discord [16], super discord [17] (see for example a recent review [18]). The notion of quantum entanglement briefly refers to nonlocal property of a global system that cannot be emulated by local descriptions (with local operation and classical communication (LOCC)). On the other hand, the quantum correlation in a broader sense may be thought of as how much quantumness one cannot access about the state of the global system by accessing only one subsystem. The quantum correlation which is not captured by entanglement measures, has been a subject of growing interest in recent years. It is important to propose feasible schemes that can distinguish classical correlations, entanglement and genuine quantum correlations present in composite quantum systems. Several proposals have come up in recent years to detect non-classical correlations. To name few proposals, it has been suggested that the quantum correlation can be quantified by measuring the expectation value of a small set

of observables on four copies of the state [19]. Even intrinsic quantum uncertainty for a single observable gives a measures of nonclassical correlations similar to that of the discord [20]. Also, quantum states with non-zero correlation can lead to a nonzero precision in the parameter estimation [21].

In interference setup, typically, one considers a particle in a pure state. If one coherently splits an incident particle and applies a unitary transformation on one arm of the interferometer, upon recombining the two paths one sees an interference. The visibility in the interference depends on  $|\langle \Psi | U | \Psi \rangle|^2$ . However, if we have mixed states, then it was not easy to define the relative phase and the visibility. In an important paper [22], it was proved that if we send a mixed state through a Mach-Zender interferometer, such that  $\rho \rightarrow \rho' = U\rho U^\dagger$  then the *relative phase* between  $\rho$  and  $\rho'$  is given by  $\text{ArgTr}(\rho U)$  and the *visibility* is defined as  $V = V(\rho, U) = |\text{Tr}(\rho U)|$ . This generalizes the notion of relative phase shift known as the Pancharatnam phase from pure states [23] to mixed states. The relative phase and the visibility for mixed states have been measured in NMR and photon interference experiments [24–26]. The notion of visibility for the mixed state has found several applications in recent years. It has been suggested that using the interferometric setup one can measure linear and non-linear functions of the density operator [27] as well as detect entanglement of unknown density operator which needs estimation of  $(d^2 - 1)$  parameters and joint measurement on  $d$  copies of (transformed) density operator [28]. This also paves the way to define the notion of interference of quantum channels [29]. Recently, it has been shown that using the notion of visibility one can define a new metric for the density operator along the unitary orbit and this gives a tighter bound on the quantum speed limit compared to other known bounds [30].

Non-classical correlation in the composite system is not only a resource, but it also plays a major role in understanding the dynamics of reduced systems of a composite system. Presence of initial correlations prohibit the description of the reduced dynamics to be completely positive map [31–36]. In

this regard methods for the detection of quantum correlation in system-environment has been proposed and it has been shown that the amount to which the evolution of the reduced state differs from the evolution of the local dephased reduced state can detect the initial correlation [37].

In this paper, we explore if it is possible to quantify the non-classical correlation and entanglement using the concepts of quantum interferometry for mixed states. We recourse to the notion of interference of mixed states and prove that the difference in the unitary average of the mixed state (squared) visibility of the density operator and its dephased counterpart is directly related to the quantum correlation measure which is defined via the measurement disturbance. Then, we show that our method of detecting quantum correlation is also applicable for noisy measurements. For pure bipartite states the average of the difference in the squared visibility before and after the measurement is related to the concurrence. Furthermore, for pure bipartite states, we prove that under local unitary evolution of one subsystem, the unitary average of the squared visibility gives the entanglement. For mixed states, using the convex roof construction, we give a new lower bound for the linear entropy version of the entanglement of formation in terms of the average visibility. In addition, we prove that for a fixed purity of the subsystem state, there is a complementarity relation between the linear entanglement of formation and the measurement disturbance. Possibly, this suggests a new difference between two kinds of quantum correlations, hitherto unnoticed. Our result makes a direct connect between two fundamental ingredients of quantum theory, namely, the quantum interference and the quantum correlation. This may constitute direct detection of entanglement including other non-classical correlations with quantum interference setup. Our method can be tested in interference experiments and the presence of non-classical correlations can be revealed.

The paper is organized as follows. In section II, we show that the difference in the average of the interference visibility for the density operator before and after dephasing is directly related to the measurement disturbance. In section III, we show how our method can detect quantum correlation with the noisy measurements. In section IV, we show that for pure bipartite states, having access to one subsystem, the unitary average of the visibility gives the concurrence. For mixed states we show that the entanglement of formation is bounded from above by the average of the interference visibility. In section V, we show that for a fixed purity of the subsystem state, there is a complementarity between entanglement of formation and the measurement disturbance. Finally, we conclude in section VI.

## II. QUANTUM CORRELATION WITH VISIBILITY

Given a bipartite state  $\rho_{AB}$  on  $\mathcal{H}_A \otimes \mathcal{H}_B$ , in general, it contains classical correlation [11], non-classical correlation such as the entanglement [3] and the statistical correlation of quantum origin that is not captured by the entanglement [10–15, 17]. We will use the quantum correlation measure based on the measurement disturbance [15]. In quantum mechan-

ics, a projective measurement on a quantum system usually disturbs the state and loses its quantumness. Similarly, for a composite system, a measurement carried out on one part (or both) usually disturbs the state thereby destroying the quantum correlation present in the system. If  $\{\Pi_i^A\}$  and  $\{\Pi_j^B\}$  are complete one-dimensional orthogonal projectors in  $\mathcal{H}_A$  and  $\mathcal{H}_B$ , then after the measurement process the density operator transforms as

$$\rho_{AB} \rightarrow \Phi(\rho_{AB}) = \sum_{i,j} (\Pi_i^A \otimes \Pi_j^B) \rho_{AB} (\Pi_i^A \otimes \Pi_j^B). \quad (1)$$

The measurement disturbance based quantum correlation is defined as [15]

$$Q(\rho_{AB}) := \|\rho_{AB} - \Phi(\rho_{AB})\|_{\text{HS}}, \quad (2)$$

where  $\|A\|_{\text{HS}} = \sqrt{\text{Tr}(A^\dagger A)}$  is the Hilbert-Schmidt norm for an operator  $A$ . Physically, this tells us the extent to which the measurement operator disturbs the composite state and gives a measure of quantumness based on the distance between the classical state closest to the original bipartite state. For classical state the quantum correlation measure is zero. One can also use a quantum correlation based on measurement disturbance where projective measurement is performed on one subsystem.

Consider a bipartite state  $\rho_{AB}$  that undergoes a unitary evolution  $\rho_{AB} \rightarrow U \rho_{AB} U^\dagger$ . If one interferes the original and the unitary transformed density operator the visibility is given by

$$V^2(\rho_{AB}, U) = |\text{Tr}(\rho_{AB} U)|^2. \quad (3)$$

Now, we perform local projective measurements on  $\rho_{AB}$  and evolve the state  $\Phi(\rho_{AB})$  under the unitary evolution. Since the measurement process destroys the quantumness, it is natural that if we interfere  $\Phi(\rho_{AB})$  with  $U \Phi(\rho_{AB}) U^\dagger$ , then there will be some change in the interference pattern, both in terms of the relative phase shift and the visibility. Now, the important question we ask is for generic unitary operators how much change does occur in the visibility of the interference. If we consider the unitary operator as a unitary matrix which has been drawn from an appropriate random matrix ensemble over the unitary group  $U(d)$ , then the unitary average of change in the visibility is directly related to the quantum correlation. Throughout our paper, we will use normalized bi-invariant Haar measure  $\mu$  over unitary matrix group  $U(d)$ .

**Theorem 1.** For any bipartite density operator  $\rho_{AB}$  on  $\mathcal{H}_A \otimes \mathcal{H}_B$ , the difference between the unitary average of the visibility before and after measurement is a measure of quantum correlation, i.e.,

$$\begin{aligned} & \int_{U(d_A d_B)} [|\text{Tr}(\rho_{AB} U)|^2 - |\text{Tr}(\Phi(\rho_{AB}) U)|^2] d\mu(U) \\ &= \frac{1}{d_A d_B} Q(\rho_{AB})^2. \end{aligned} \quad (4)$$

*Proof.* The average of the (squared) visibility of quantum interference for the mixed states for generic unitary operators

can be expressed as

$$\begin{aligned} \int_{U(d)} |\text{Tr}(\rho_{AB}U)|^2 d\mu(U) &= \text{Tr} \left( \rho_{AB}^{\otimes 2} \int_{U(d)} U \otimes U^\dagger d\mu(U) \right) \\ &= \text{Tr}(\rho_{AB}^{\otimes 2} M), \end{aligned}$$

where  $M = \int_{U(d)} U \otimes U^\dagger d\mu(U)$ . First we prove that

$$M = \int_{U(d)} U \otimes U^\dagger d\mu(U) = \frac{F}{d}, \quad (5)$$

where  $F$  is the swap operator defined as  $F = \sum_{i,j} |ij\rangle\langle ji|$  and  $d = d_A d_B$ .

First, we note that  $M$  is self-adjoint by the property of the Haar-measure. From the fact that  $\text{Tr}((A \otimes B)F) = \text{Tr}(AB)$ , we have  $\text{Tr}(MF) = d$ . Since the Haar-measure is left-regular, it follows that  $(V \otimes I)M(I \otimes V^\dagger) = M$  for all  $V \in U(d)$ . By taking traces over both sides, we have  $\text{Tr}(M) = \text{Tr}(M(V \otimes V^\dagger))$ . Now, by taking integrals over both sides, we have  $\text{Tr}(M) = \text{Tr}(M^2)$ . By the Cauchy-Schwartz inequality, we get

$$d^2 = (\text{Tr}(MF))^2 \leq \text{Tr}(M^2)\text{Tr}(F^2) = d^2 \text{Tr}(M),$$

which implies that  $\text{Tr}(M) \geq 1$ . In what follows, we show that  $\text{Tr}(M) = 1$ . By the definition of  $M$ , we have

$$\text{Tr}(M) = \langle\langle I_d | \int_{U(d)} |U\rangle\rangle \langle\langle U | d\mu(U) | I_d \rangle\rangle, \quad (6)$$

where  $|X\rangle\rangle := \sum_{i,j} X_{ij} |ij\rangle$  for a matrix  $X = \sum_{i,j} X_{ij} |i\rangle\langle j|$ . Now, define a unital quantum channel  $\Gamma$  as follows:

$$\Gamma := \int_{U(d)} \text{Ad}_U d\mu(U).$$

Thus, we have  $\Gamma(X) = \text{Tr}(X) \frac{I_d}{d}$ . By the Choi-Jamiołkowski isomorphism, it follows that

$$J(\Gamma) = (\Gamma \otimes \text{id})(|I_d\rangle\rangle\langle\langle I_d|) \quad (7)$$

$$= \int_{U(d)} |U\rangle\rangle\langle\langle U | d\mu(U). \quad (8)$$

For the completely depolarizing channel  $\Gamma(X) = \text{Tr}(X) \frac{I_d}{d}$ , we already know that  $J(\Gamma) = \frac{1}{d} I_d \otimes I_d$ . Then, it follows that

$$\int |U\rangle\rangle\langle\langle U | d\mu(U) = \frac{1}{d} I_d \otimes I_d. \quad (9)$$

Finally, we have  $\text{Tr}(M) = 1$ . This indicates that the Cauchy-Schwartz inequality is saturated, and moreover the saturation happens if and only if  $M = \lambda F$  for  $\lambda$  constant. By taking traces over both sides, we have  $\lambda = \frac{1}{d}$ . The desired identity is obtained.

Therefore, by (5), the expression for the visibility of quantum interference for the mixed states averaged over all unitary group using the Haar measure is given by

$$\int_{U(d_A d_B)} |\text{Tr}(\rho_{AB}U)|^2 d\mu(U) = \frac{1}{d_A d_B} \text{Tr}(\rho_{AB}^2). \quad (10)$$

This shows that for generic unitary evolutions of quantum system, it is the purity of the density operator that ultimately decides the visibility in the interference. Since the measurement disturbance  $Q(\rho_{AB})^2 = \text{Tr}(\rho_{AB}^2) - \text{Tr}(\Phi(\rho_{AB})^2)$ , we have the proof.  $\square$

Therefore, the unitary average of the difference in the (squared) visibility before and after the measurement performed on the composite system is given by the quantum correlation based on measurement disturbance.

In our proof of Theorem 1, we have used global unitary and have shown that the unitary average of the interference visibility before and after a complete measurement is related to quantum correlation. However, we can also have the same result when the interference visibility is obtained under local unitary and average over local unitary groups. Detection of quantum entanglement and quantum correlation using local operations is important when we do not have access to the whole system to reveal these correlations present in the composite system. Therefore, our method can help in detection of quantum correlation with access to local subsystems only. In fact, we can prove that for any bipartite state  $\rho_{AB}$ , the unitary average of visibility under local unitary satisfies

$$\begin{aligned} &\int_{U(d_A)} \int_{U(d_B)} |\text{Tr}(\rho_{AB}(U \otimes V))|^2 d\mu(U) d\mu(V) \\ &= \frac{1}{d_A d_B} \text{Tr}(\rho_{AB}^2). \end{aligned} \quad (11)$$

To prove this consider the spectral decomposition of  $\rho_{AB}$ , with  $\rho_{AB} = \sum_j \lambda_j |\Phi_j\rangle\langle\Phi_j|$ , where  $\lambda_j$ 's and  $|\Phi_j\rangle$ 's are the eigenvalues and the eigenvectors of  $\rho_{AB}$ , respectively. Now we see that there exist  $d_B \times d_A$  matrices  $Y_j$  such that  $|\Phi_j\rangle = |Y_j\rangle\rangle$ . This suggests that we can write

$$\begin{aligned} &|\text{Tr}(\rho_{AB}(U \otimes V))|^2 = \sum_{i,j} \lambda_i \lambda_j \\ &\times \text{Tr} \left[ (Y_i^\dagger \otimes Y_j^\dagger)(U \otimes U^\dagger)(Y_i \otimes Y_j)(V^\dagger \otimes (V^\dagger)^\dagger) \right]. \end{aligned}$$

Thus, we have

$$\begin{aligned} &\int_{U(d_A)} \int_{U(d_B)} |\text{Tr}(\rho_{AB}(U \otimes V))|^2 d\mu(U) d\mu(V) \\ &= \frac{1}{d_A d_B} \sum_{i,j} \lambda_i \lambda_j \text{Tr}((Y_i \otimes Y_j)^\dagger F_{AA} (Y_i \otimes Y_j) F_{BB}), \end{aligned}$$

where  $F_{AA}$  is the swap operator on  $\mathbb{C}^{d_A} \otimes \mathbb{C}^{d_A}$ ,  $F_{BB}$  is the swap operator on  $\mathbb{C}^{d_B} \otimes \mathbb{C}^{d_B}$ . Taking orthonormal base  $|\mu\rangle$  and  $|m\rangle$  of  $\mathbb{C}^{d_A}$  and  $\mathbb{C}^{d_B}$ , respectively, gives rise to  $F_{AA} = \sum_{\mu,\nu=1}^{d_A} |\mu\nu\rangle\langle\nu\mu|$ ,  $F_{BB} = \sum_{m,n=1}^{d_B} |mn\rangle\langle nm|$ . By substituting both these operators into the above expression, it follows that

$$\begin{aligned} &\int_{U(d_A)} \int_{U(d_B)} |\text{Tr}(\rho_{AB}(U \otimes V))|^2 d\mu(U) d\mu(V) \\ &= \frac{1}{d_A d_B} \sum_{i,j} \lambda_i \lambda_j \text{Tr}(Y_i Y_j^\dagger) \text{Tr}(Y_i^\dagger Y_j), \end{aligned}$$

which is equal to  $\frac{1}{d_A d_B} \text{Tr}(\rho_{AB}^2)$ .

Therefore, having access to local subsystem under local measurement, one can define the measurement disturbance and the change in the interference visibility can be related to this quantum correlation.

### III. VISIBILITY AND QUANTUM CORRELATION WITH NOISY MEASUREMENT DISTURBANCE

In previous section, we have shown that on the average the change in the visibility of the density operator under going unitary evolution before and after a complete measurement, is directly related to the quantum correlation called the measurement disturbance. We can ask, how robust is our result to noisy measurements. Specifically, suppose we define a quantum correlation measure based on the noisy measurement which causes partial collapse of the subsystem. Can we still detect the quantum correlation with change in the visibility? We can model a noisy measurement in which, with probability  $\epsilon$ , the complete measurement is performed and with a probability  $(1 - \epsilon)$  no measurement occurs. A complete measurement is the special case of the noisy measurement, when  $\epsilon = 1$ . This can also be thought of as a quantum channel  $\Phi_{(\epsilon)}$  whose action is defined as

$$\Phi_{(\epsilon)}(\rho_{AB}) := \epsilon\Phi(\rho_{AB}) + (1 - \epsilon)\rho_{AB}, \quad (12)$$

where  $\epsilon \in [0, 1]$  and  $\text{Tr}[\Phi_{(\epsilon)}(\rho)] = 1$ . This channel is nothing but a convex combination of a channel that keeps the state undisturbed and a channel that implements the strong measurement. Here,  $\Phi_{(\epsilon)}$  may act on one subsystem or both the subsystems. The noisy measurement can arise in the case of imperfect detectors and in the case of partial measurements. We will show that by comparing the visibility of the density operator undergoing unitary evolution before and after a measurements that may give incomplete information, on the average we can detect quantum correlation.

Now for the noisy measurement, we define the quantum correlation measure as

$$Q_{(\epsilon)}(\rho_{AB}) := \|\rho_{AB} - \Phi_{(\epsilon)}(\rho_{AB})\|_{\text{HS}}. \quad (13)$$

From the definition, we can see that the noisy-measurement disturbance quantum correlation is related to the ideal measurement disturbance quantum correlation as  $Q_{\epsilon}(\rho_{AB}) = \epsilon Q(\rho_{AB})$ . If  $\epsilon < 1$ , then noisy-measurement disturbance quantum correlation is less than the measurement disturbance quantum correlation. Below, we will show that for any bipartite density operator, the difference between the unitary average of the visibility before and after a noisy measurement is directly related to  $Q(\rho_{AB})$ , i.e.,

$$\begin{aligned} & \int [|\text{Tr}(\rho_{AB}U)|^2 - |\text{Tr}(\Phi_{(\epsilon)}(\rho_{AB})U)|^2] d\mu(U) \\ &= \epsilon(2 - \epsilon) \frac{1}{d_A d_B} Q(\rho_{AB})^2. \end{aligned} \quad (14)$$

This shows that our method of witnessing quantum correlation with visibility is robust against noisy measurements. As long as the composite system is disturbed by measurement

(whether by a complete von Neumann measurement or noisy measurement), by looking at the difference of the visibility of the density operator before and after the measurement, averaged over generic unitary operators, we can capture the non-classical correlation beyond entanglement.

### IV. VISIBILITY AS WITNESS OF ENTANGLEMENT OF PURE AND MIXED STATES

One of the prime area of research in quantum information is how to detect entanglement in composite quantum systems. Once we make sure that the states at disposal are indeed entangled then we can use them for various quantum information processing tasks. Here, we will show that for pure bipartite entangled states, having access to a local system one can detect entanglement in interferometry visibility. Consider a general pure entangled state

$$|\Psi\rangle_{AB} = \sum_i \sqrt{\lambda_i} |\psi_i\rangle_A \otimes |\phi_i\rangle_B, \quad (15)$$

where  $\lambda_i$  are the Schmidt coefficients, and  $|\psi_i\rangle_A \in \mathcal{H}_A, |\phi_i\rangle \in \mathcal{H}_B$  are the local Schmidt bases. Let  $|\Psi\rangle_{AB} \rightarrow (U \otimes I)|\Psi\rangle_{AB}$ , and this induces a local unitary evolution for the subsystem  $A$ , i.e.,  $\rho_A \rightarrow U\rho_A U^\dagger$ . Now, the interference visibility for the subsystem  $A$  averaged over unitary group is given by

$$\int_{U(d_A)} |\text{Tr}(\rho_A U)|^2 d\mu(U) = 1/d_A \text{Tr}(\rho_A^2). \quad (16)$$

For pure bipartite entangled state, we can consider the linear entropy  $S_L(\rho_A) = S_L(\rho_B) = E(\Psi)$  as a measure of entanglement, where  $E(\Psi) = 1 - \text{Tr}(\rho_A^2)$ . Hence, the pure state entanglement measure and the unitary average of the (squared) interference visibility are directly related. This can be neatly expressed as

$$E(\Psi) = \left(1 - d_A \int |\text{Tr}(\rho_A U)|^2 d\mu(U)\right). \quad (17)$$

If we use the concurrence  $C(\Psi)$  as a measure of entanglement [4, 38], where  $C(\Psi) = \sqrt{2(1 - \text{Tr}(\rho_A^2))}$ , then the unitary average of the (squared) interference visibility can be expressed as  $C(\Psi)^2 = 2(1 - d_A \int |\text{Tr}(\rho_A U)|^2 d\mu(U))$ .

In the actual experiments, one need not run the interference over all unitaries. Because of our result in section I, we have the unitary average as a swap operator that one needs to apply on two copies of the same system. This is consistent with earlier method of direct detection of entanglement and measurement of linear and non-linear functions of density operators [28].

For mixed states, via the convex-roof construction, we can define the entanglement measure (entanglement of formation) [5, 6]. It is hard to find analytic expressions for the entanglement of formation and hence lower and upper bounds are very useful [39–41]. For two-qubits this is a monotonically increasing function of the concurrence, and it is possible to

have a closed formula using the concurrence [4]. However, in general it is not easy. We will give a new lower bound for the entanglement of formation in terms of the average visibility. Consider a mixed state with pure state decomposition

$$\rho_{AB} = \sum_j p_j |\Psi_j\rangle\langle\Psi_j| \quad (18)$$

with  $\sum_j p_j = 1$  and  $|\Psi_j\rangle$  are not orthogonal in general.

In quantum information the linear entropy is an useful concept and it has been exploited in the monogamy of concurrence and concurrence of assistance for multi-particle systems [42]. If we use the linear entropy and its convex-roof generalization as a measure of entanglement of  $\rho_{AB}$ , then we have the linear entanglement of formation defined as

$$E_F(\rho_{AB}) = \min_{\{p_j, \Psi_j\}} \sum_j p_j E(\Psi_j), \quad (19)$$

where the minimum is taken over all decomposition of  $\rho_{AB}$ ,  $E(\Psi_j) = 1 - \text{Tr}(\rho_{A_j}^2)$  and  $\rho_{A_j} = \text{Tr}_B(|\Psi_j\rangle\langle\Psi_j|)$ . From the definition, the linear entanglement of formation satisfies

$$E_F(\rho_{AB}) \leq \sum_j p_j (1 - \text{Tr}(\rho_{A_j}^2)). \quad (20)$$

Now let  $\rho_{AB}$  undergoes a local unitary evolution, where the unitary acts on the subsystem  $A$ , i.e.,  $\rho_{AB} \rightarrow \rho'_{AB} = (U \otimes I_B) \rho_{AB} (U^\dagger \otimes I_B)$ . Then, the visibility function for  $\rho_{AB}$  under the local unitary can be defined as  $V^2(\rho_{AB}, U \otimes I_B) = |\text{Tr}(\rho_{AB} U)|^2$ , which is no more than  $\sum_j p_j V^2(\rho_{A_j}, U)$ . By denoting  $\bar{V}^2 = \int_U V^2 d\mu(U)$ , the above inequality reads as  $\bar{V}^2 \leq \sum_j p_j \bar{V}_j^2$  with  $\bar{V}_j^2 = \int_U |\text{Tr}(\rho_{A_j} U)|^2 d\mu(U)$ . Therefore, from (20), we obtain

$$E_F(\rho_{AB}) \leq 1 - d_A \bar{V}^2. \quad (21)$$

Thus, the linear entanglement of formation for bipartite mixed state is upper bounded by a quantity that depends on the average interference visibility. Looking at (21), one can also interpret this as a complementary relation between the entanglement of formation and the average visibility. In any bipartite state if the average visibility of the local subsystem undergoing unitary evolution is less then they will share more entanglement. In the subsequent section, we dwell on this complementarity relation more and reveal a new kind of complementarity between entanglement and the measurement induced disturbance.

## V. COMPLEMENTARITY OF ENTANGLEMENT AND MEASUREMENT DISTURBANCE

For pure bipartite states, we know that entanglement measures and quantum correlation measures based on the measurement disturbance coincide. However, for mixed state there is no quantitative connection between quantum entanglement and quantum correlation that supposedly captures something beyond entanglement. The only relation that we

know is the Koashi-Winter relation [44] that connects the entanglement of formation and quantum correlation across different partitions of a tripartite density operator  $\rho_{ABC}$ . Here, we will show that for any bipartite mixed state  $\rho_{AB}$  the linear entanglement of formation and quantum correlation respects a complementarity relation for a given purity of the subsystem state. This is first ever direct quantitative connection between the entanglement and the quantum correlation for any bipartite state across the same partition.

Let us consider the quantum correlation measure based on the local measurement performed on the subsystem  $A$ . If  $\{\Pi_i^A\}$  is a set of complete one-dimensional orthogonal projectors on  $\mathcal{H}_A$ , then after the measurement process the density operator transforms as

$$\begin{aligned} \rho_{AB} \rightarrow \rho'_{AB} &= \Phi(\rho_{AB}) = \sum_i (\Pi_i^A \otimes I^B) \rho_{AB} (\Pi_i^A \otimes I^B) \\ &= \sum_i p_i |i\rangle\langle i| \otimes \rho_{B|i}. \end{aligned} \quad (22)$$

Note that for any bipartite density operator  $\rho_{AB}$  we have the relation between the global and the local purity as given by [43]

$$d_B^{-1} \leq \frac{\text{Tr}(\rho_{AB}^2)}{\text{Tr}(\rho_A^2)} \leq d_B$$

Using (21) and the above equation we can derive the complementarity relation. First, note that for the post-measured state, we have  $\text{Tr}(\rho_{AB}'^2) \geq \text{Tr}(\rho_B'^2)/d_A = \text{Tr}(\rho_B^2)/d_A$  (as local measurement on the subsystem  $A$  does not change the purity of  $B$ ). Therefore, we have the following inequality

$$E_F(\rho_{AB}) + \frac{1}{d_B} Q(\rho_{AB}) + \frac{1}{d_A d_B} P(\rho_B) \leq 1, \quad (23)$$

where  $P(\rho_B) = \text{Tr}(\rho_B^2)$  is the purity of the subsystem  $B$ . This shows that the total quantum correlation (the entanglement of formation plus the measurement disturbance) and the subsystem purity can be complementary to each other. If in the bipartite state, the subsystem state is of less purity, then the total quantum correlations can be more in that state. For separable states, we have similar complementary relation between the subsystem purity and the measurement disturbance. Another consequence of our new relation is that for a fixed purity of the subsystem state  $\rho_B$ , the above relation shows yet another kind of complementarity, namely, there can be trade-off between pure non-local quantum correlation such as the linear entanglement of formation and the non-classical correlation such as the measurement disturbance. This suggests that the entanglement measure and the non-classical correlation based on measurement disturbance can be genuinely different in nature. Whether such complementarity relation is a generic feature of entanglement and other non-classical correlations is an open question. This deserves further exploration in future.

## VI. CONCLUSIONS

In quantum theory, interference and non-classical correlations are two fundamental concepts. In the era of quantum information they play major roles in information processing tasks. Whether it is about speed-ups in quantum computer or advantages in quantum communications one always looks for these non-classical effects to harness the true power of complex quantum systems. In this paper, we have shown that these two concepts are connected in a quantitative manner. We have shown that the difference in the squared visibility for the density operator before and after a complete measurement, averaged over all unitary evolutions, is directly related to the quantum correlation measure based on the measurement disturbance. For pure bipartite states, having access to one subsystem, the unitary average of the visibility gives the concurrence. For mixed states we have shown that the linear entanglement of formation is bounded from above

by the average of interference visibility. Furthermore, we have shown that for a fixed purity of the subsystem state, there is a complementarity relation between the entanglement of formation and the measurement disturbance. This brings out a fundamental difference between these two kinds of quantum correlations, hitherto unnoticed. Our proposal can be tested in interference experiments and the presence of quantum correlation and entanglement can be revealed.

*Acknowledgement.*—LZ is grateful for financial support from Natural Science Foundations of China (No.11301124). AKP thanks Department of Mathematics, Zhejiang University for supporting his visit as a K. P. Chair Professor during which this work is carried out. JW is also supported by Natural Science Foundations of China (1171301 and 10771191) and the Doctoral Programs Foundation of Ministry of Education of China (J20130061).

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